

Consciousness and Free Will Under Deterministic Laws

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Interface. Let $g \in B := \{-1, 1\}$ be the control direction and $x = (i, m) \in X := B^2$ the state of separately preparable registers, command i and model m . For $q, t \in B$ define the model output, readout, model inversion and diagnostic by

$$f_m(t) := mt = f_m^{-1}(t), \quad o(i, m) := f_g(i) = gi, \quad D_q(i, m) := (f_m^{-1}(q), m), \quad \Theta := (o, oD_{+1}).$$

The readout o leaves the state unchanged. Θ records o before and after one D_{+1} . Maps compose right to left. Define the Stone dual $\mathcal{D}(X) := \text{Hom}_{\text{Bool}}(\mathcal{P}(X), \{0, 1\})$ and the evaluation $\eta(x)(A) := \mathbf{1}_A(x)$.

Grounding. Let $\mathcal{O}$ be the occurrence tokens of a fixed context and $\mathcal{E} \subseteq \mathcal{O}$ the experienced ones. Let $y_0 \in Y$. Assume denotation maps and a reference-preserving recognition map satisfy

$$d_0 : Y \rightarrow \mathcal{O}, \quad d : X \rightarrow \mathcal{O}, \quad C : Y \rightarrow X, \quad dC = d_0, \quad d_0(y_0) = a \in \mathcal{E}.$$

Theorem 1 (Consciousness). Two readouts force one subjective experience of a grounded occurrence.

The diagnostic Θ determines the state and the truth value of every predicate. Every deterministic realization of the interface has at least four complete retained states. Define $b := \Theta(Cy_0) \in B^2$, $x_b := (gb_1, gb_2)$ and $\hat{d} := d\eta^{-1}$. Then

$$\mathcal{V}_b := \{v \in \mathcal{D}(X) : v(o^{-1}\{b_1\}) = v(D_{+1}^{-1}o^{-1}\{b_2\}) = 1\} = \{v_b\}, \quad v_b = \eta(x_b), \quad \hat{d}(v_b) = a \in \mathcal{E}.$$

η conjugates every update of X and preserves the output of every finite update sequence.

Proof. For $s, t \in B$ direct substitution gives

$$\Theta(i, m) = (gi, gm), \quad \Theta^{-1}(s, t) = (gs, gt), \quad o^{-1}\{s\} \cap D_{+1}^{-1}o^{-1}\{t\} = \{(gs, gt)\}.$$

These level sets generate $\mathcal{P}(X)$. For $v \in \mathcal{D}(X)$ finite additivity gives

$$\sum_{x \in X} v(\{x\}) = 1, \quad v(A) = \sum_{x \in A} v(\{x\}), \quad v = \eta(x_v) \text{ for a unique } x_v \in X, \quad v \in \mathcal{V}_b \iff \Theta(x_v) = b.$$

So η is bijective and $\mathcal{V}_b = \{\eta(x_b)\}$. Since Θ is injective, $x_b = \Theta^{-1}(b) = Cy_0$ and $\hat{d}(v_b) = d(Cy_0) = d_0(y_0) = a$. For $T : X \rightarrow X$ define $T^*A := T^{-1}(A)$, $\hat{T}(v) := v \circ T^*$ and $\hat{o} := o\eta^{-1}$. For $k \in \mathbb{Z}_{\geq 0}$ and updates $T_1, \dots, T_k$

$$\hat{T}\eta(x)(A) = \mathbf{1}_{T^{-1}(A)}(x) = \eta(Tx)(A), \quad \hat{T} = \eta T \eta^{-1}, \quad \hat{o}\hat{T}_k \cdots \hat{T}_1 \eta = oT_k \cdots T_1.$$

A deterministic realization on complete retained states Z has $e : X \rightarrow Z$, $r : Z \rightarrow B$ and $U : Z \rightarrow Z$ with $re = o$ and $rUe = oD_{+1}$. Both readouts factor through e . Therefore e is injective and $|Z| \geq 4$. The interface itself is a realization with four retained states and attains the bound. 

Theorem 2 (Free Will). Conscious intent affects physical action under deterministic laws.

Assume η identifies the complete physical and experiential presentations of the subject realized on X and intertwines its interventions. Assume the felt intention $\mathbf{l}$ and the represented aim $\mathbf{A}$ satisfy $\mathbf{l}\eta(i, m) = i$ and $\mathbf{A}\eta(i, m) = mi$. X carries a probe P and appraisals R_ρ ($\rho \in \mathbb{R}$) with $R_{2q} = D_q$. Both are total deterministic maps. Define $X_g := B \times \{g\}$, $\Gamma := PD_{-1}PD_{+1}$, $E_r(i, m) := (r, m)$ and $x_\sigma := (\sigma, -g)$. For $q, r, \sigma \in B$, $\rho \in \mathbb{R}$, $(i, m) \in X$ and $v \in \mathcal{D}(X)$,

$$\begin{aligned} \Gamma(X) &\subseteq X_g, \quad P|_{X_g} = \text{id}_{X_g}, \quad R_\rho(X_g) \subseteq X_g; \\ |\rho| \leq \frac{3}{2} &\implies R_\rho = \text{id}_X, \quad R_\rho(i, m) \neq (i, m) \iff -2\rho mi > 3; \\ \mathbf{A}(\hat{D}_q \hat{\Gamma} v) &= \hat{o} \hat{D}_q \hat{\Gamma} v = q, \quad (oD_q E_r P(x_\sigma), oD_q E_r(x_\sigma)) = (\sigma q, -q); \\ \mathbf{l}(\hat{E}_r v) &= r, \quad \hat{o} \hat{E}_{+1} v - \hat{o} \hat{E}_{-1} v = 2g \neq 0. \end{aligned}$$

At the grounded state v_b of Theorem 1,

$$\rho_j = -jb_1 b_2 \quad (j = 1, 2) \implies (\hat{o} \hat{R}_{\rho_1}(v_b), \hat{o} \hat{R}_{\rho_2}(v_b)) = (b_1, -b_1).$$

Proof. Define P and R_ρ by the rules below. P senses g when $i = 1$ and R_ρ keeps i at ties.

$$P(i, m) := \left(i, \frac{1+i}{2}g + \frac{1-i}{2}m\right), \quad R_\rho(i, m) := (\beta_\rho, m), \quad \beta_\rho \in \arg \max_{\beta \in B} s_\rho(\beta | i, m), \quad s_\rho(\beta | i, m) := \rho m \beta + 3\mathbf{1}_{\{\beta=i\}}.$$

Switching the command changes the score by

$$\Delta_\rho(i, m) = -2\rho mi - 3, \quad R_\rho(i, m) = (i(1 - 2\mathbf{1}_{\{\Delta_\rho > 0\}}), m), \quad R_{2q}(i, m) = (qm, m) = D_q(i, m).$$

The bound $\Delta_\rho \leq 2|\rho| - 3$ shows that appraisals with $|\rho| \leq 3/2$ never change the state. Substitution gives

$$\Gamma(i, 1) = (-g, g), \quad \Gamma(i, -1) = (1, g), \quad P|_{X_g} = \text{id}_{X_g}, \quad R_\rho(X_g) \subseteq X_g, \quad (\mathbf{A}\eta)D_q(i, g) = oD_q(i, g) = g(qg) = q;$$

$$x_\sigma \xrightarrow{P} (\sigma, \sigma g) \xrightarrow{E_r} (r, \sigma g) \xrightarrow{D_q} (\sigma q g, \sigma g) \xrightarrow{o} \sigma q, \quad E_r x_\sigma = (r, -g) \implies oD_q E_r x_\sigma = -q.$$

The last row follows from $\mathbf{l}\hat{E}_r \eta(i, m) = r$ and $\hat{o}\hat{E}_r \eta(i, m) = gr$. Setting the felt intention to r produces the output gr and the two settings differ by $2g$. At x_b the margins $\Delta_{\rho_j} = 2j - 3$ equal -1 and 1 . R_{ρ_1} therefore keeps the command gb_1 and outputs b_1 while R_{ρ_2} reverses it and outputs $-b_1$. Theorem 1 gives the same two outputs at v_b . 